

ΜΑΘΗΜΑΤΙΚΟ ΤΥΠΟΛΟΓΙΟ

Σελίδα 1 από 1 - 1/1



Σταθερές.

π	=	03,14159 26535 89793 23846 2643...
e	=	02,71828 18284 59045 23536 0287...
e^π	=	23,14069 26327 79269 006...
π^e	=	22,45915 77183 61045 47342 715...
e^e	=	15,15426 22414 79264 190...
$\sqrt{2}$	=	01,41421 35623 73095 0488...
$\sqrt{3}$	=	01,73205 08075 68877 2935...
$\sqrt{5}$	=	02,23606 79774 99789 6964...
$\sqrt{e}$	=	01,64872 12707 00128 1468...
$\sqrt{\pi}$	=	01,77245 38609 05516 02729 8167...
$\log_2$	=	00,30102 99956 63981 19521 37389...
$\log_3$	=	00,47712 12547 19662 43729 50279...
$\log e$	=	00,43429 44819 03251 82765...
$\log \pi$	=	00,49714 98726 94133 85435 12683...
$\ln 2$	=	00,69314 71805 59945 30941 7232...
$\ln 3$	=	01,09861 22886 68109 69139 5245...
$\ln 10$	=	02,30258 50929 94045 68401 7991...
$\ln \pi$	=	01,14472 9886...

Μαθηματική λογική.

P	q	$\bar{p}$	$p \wedge q$	$p \vee q$	$p \underline{\vee} q$	$p \Rightarrow q$	$p \Leftrightarrow q$
α	α	ψ	α	α	ψ	α	α
α	ψ	ψ	ψ	α	α	ψ	ψ
ψ	α	α	ψ	α	α	α	ψ
ψ	ψ	α	ψ	ψ	ψ	α	α

- $(p \Rightarrow q) \Leftrightarrow (\bar{q} \Rightarrow \bar{p})$

Σύνολλα.

- $A \subseteq B \Leftrightarrow [x \in A \Rightarrow x \in B]$
- $A \subset B \Leftrightarrow [(x \in A \Rightarrow x \in B) \wedge \exists x \in B : x \notin A]$
 $\Leftrightarrow [A \subseteq B \wedge \exists x \in B : x \notin A]$
- $A=B \Leftrightarrow [(A \subseteq B) \wedge (B \subseteq A)]$
- $A \cap B \equiv \{x : x \in A \wedge x \in B\}.$
- $A \cup B \equiv \{x : x \in A \vee x \in B\}.$
- $A-B \equiv \{x \in A : x \notin B\}.$
- $A^c \equiv U-A = \{x \in U : x \notin A\}.$
- $A \dot{+} B \equiv (A-B) \cup (B-A).$
- $A \cap \emptyset = \emptyset$ και $A \cup \emptyset = A, \forall A.$
- $A \cap A = A$ και $A \cup A = A, \forall A.$
- $A \cap U = A$ και $A \cup U = U, \forall A.$
- $A \cap (B \cap \Gamma) = (A \cap B) \cap \Gamma$ και $A \cup (B \cup \Gamma) = (A \cup B) \cup \Gamma \forall A, B, \Gamma$
- $A \cap B = B \cap A$ και $A \cup B = B \cup A, \forall A, B$
- $A \cup (B \cap \Gamma) = (A \cup B) \cap (A \cup \Gamma), \forall A, B, \Gamma$
- $A \cap (B \cup \Gamma) = (A \cap B) \cup (A \cap \Gamma), \forall A, B, \Gamma$

Συμβολισμοί και ιδιότητές τους.

- $v! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot v$ με $v \in \mathbb{N}^*$ και $0! = 1.$
- $\binom{v}{\kappa} = \frac{v!}{\kappa!(v-\kappa)!}, \quad v, \kappa \in \mathbb{N}.$
- $\binom{v}{\kappa} + \binom{v}{\kappa+1} = \binom{v+1}{\kappa+1}, \quad v, \kappa \in \mathbb{N}.$
- $\sum_{i=1}^v x_i = x_1 + x_2 + x_3 + \dots + x_v.$
- $\sum_{i=1}^v \lambda x_i = \lambda \cdot \sum_{i=1}^v x_i$
- $\sum_{i=1}^v (x_i + y_i) = \sum_{i=1}^v x_i + \sum_{i=1}^v y_i$
- $\sum_{i=1}^v x = v \cdot x.$

Αξιοσημείωτες ταυτότητες.

- $(\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2$.
- $(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2$.
- $(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3=\alpha^3+\beta^3+3\alpha\beta(\alpha+\beta)$.
- $(\alpha-\beta)^3=\alpha^3-3\alpha^2\beta+3\alpha\beta^2-\beta^3=\alpha^3-\beta^3-3\alpha\beta(\alpha-\beta)$.
- $(\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\beta\gamma+2\gamma\alpha$.
- $(\alpha+\beta+\gamma)^3=\alpha^3+\beta^3+\gamma^3+3(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$.
- $(\alpha+\beta)(\alpha-\beta)=\alpha^2-\beta^2$.
- $(x-\alpha)(x-\beta)=x^2-(\alpha+\beta)x+\alpha\beta$.
- $\alpha^v-\beta^v=(\alpha-\beta)\cdot(\alpha^{v-1}+\alpha^{v-2}\beta+\alpha^{v-3}\beta^2+\dots+\alpha\beta^{v-2}+\beta^{v-1}), \forall v\in\mathbf{N}^*$.
- $\alpha^v+\beta^v=(\alpha+\beta)\cdot(\alpha^{v-1}-\alpha^{v-2}\beta+\alpha^{v-3}\beta^2-\dots-\alpha\beta^{v-2}+\beta^{v-1}), \forall v\in\mathbf{N}^* \therefore v=2\kappa+1$
- $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma=(\alpha+\beta+\gamma)\cdot(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)$.
- $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma=\frac{1}{2}(\alpha+\beta+\gamma)\cdot[(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2]$.
- $(\alpha+\beta)^v=\sum_{\kappa=0}^v\binom{v}{\kappa}\cdot\alpha^{v-\kappa}\cdot\beta^{\kappa}$

Χρήσιμες ανισότητες.

- $x^2\geq 0, \forall x\in\mathbf{R}$.
- $\alpha^2+\beta^2\geq 2\alpha\beta$ και $\alpha^2+\beta^2\geq -2\alpha\beta \forall \alpha,\beta\in\mathbf{R}$.
- $\alpha^2+\beta^2\geq \alpha\beta$ και $\alpha^2+\beta^2\geq -\alpha\beta \forall \alpha,\beta\in\mathbf{R}$.
- $\alpha^2+\beta^2+\gamma^2\geq \alpha\beta+\beta\gamma+\gamma\alpha. \forall \alpha,\beta,\gamma\in\mathbf{R}$.
- $(1+\alpha)^v\geq 1+v\cdot\alpha, \alpha\geq -1 \forall v\in\mathbf{R}$. (Bernoulli)

Απόλυτη τιμή.

- $|\alpha|=\begin{cases} \alpha, & \text{αν } \alpha\geq 0 \\ -\alpha, & \text{αν } \alpha<0 \end{cases}$
- $|\alpha|\geq 0 \quad \forall \alpha\in\mathbf{R}$.
- $|\alpha|^2=\alpha^2 \quad \forall \alpha\in\mathbf{R}$.
- $-|\alpha|\leq \alpha\leq |\alpha| \quad \forall \alpha\in\mathbf{R}$.
- $|x|=\alpha \Leftrightarrow x=\alpha \text{ ή } x=-\alpha$.
- $|x|\leq \varepsilon \Leftrightarrow -\varepsilon\leq x\leq \varepsilon$.
- $|x|\geq \alpha \Leftrightarrow \text{ή } x\leq -\alpha \text{ ή } x\geq \alpha$.
- $|\alpha\cdot\beta|=|\alpha|\cdot|\beta| \quad \forall \alpha,\beta\in\mathbf{R}$.
- $\left|\frac{\alpha}{\beta}\right|=\frac{|\alpha|}{|\beta|}, \quad \forall \alpha\in\mathbf{R}, \forall \beta\in\mathbf{R}^*$.
- $||\alpha| - |\beta|| \leq |\alpha\pm\beta| \leq |\alpha| + |\beta|, \quad \forall \alpha,\beta\in\mathbf{R}$.

Δευτεροβάθμιο Τριώνυμο.

- Τριώνυμο $\pi(x)=ax^2+\beta x+\gamma$, $a \neq 0$.
- Διακρίνουσα $\Delta=\beta^2-4\cdot a\cdot \gamma$.
- Ρίζες $x_{1,2}=\frac{-\beta \pm \sqrt{\Delta}}{2\cdot a}$.
- $S=x_1+x_2=-\frac{\beta}{a}$ και $P=x_1\cdot x_2=\frac{\gamma}{a}$.

Αριθμητική Πρόοδος.

- Ορισμός $a_{v+1}=a_v+\omega$, $v=1,2,3,\dots$ ω : διαφορά.
- Νιοστός όρος $a_v=a_1+(v-1)\cdot \omega$, $v \in \mathbf{N}^*$.
- Άθροισμα n πρώτων όρων $\Sigma_v=\frac{a_1+a_n}{2} \cdot n=\frac{2a_1+(n-1)\cdot \omega}{2} \cdot n$
- β : αριθμητικός μέσος των $a, \gamma \Leftrightarrow 2\beta=a+\gamma$.

Γεωμετρική Πρόοδος.

- Ορισμός $a_{v+1}=a_v\cdot \lambda$, $v=1,2,3,\dots$ λ : λόγος.
- Νιοστός όρος $a_v=a_1\lambda^{v-1}$, $v \in \mathbf{N}^*$.
- Άθροισμα n πρώτων όρων
$$\Sigma_v=\begin{cases} \frac{a_v\lambda-a_1}{\lambda-1}=\frac{a_1(\lambda^n-1)}{\lambda-1} & \text{αν } \lambda \neq 1 \\ n\cdot a_1 & \text{αν } \lambda=1 \end{cases}$$
- Άθροισμα άπειρων όρων (αν $|\lambda|<1$) $\Sigma_\infty=\frac{a_1}{1-\lambda}$
- β : γεωμετρικός μέσος των $a, \gamma \Leftrightarrow \beta^2=a\cdot \gamma$.

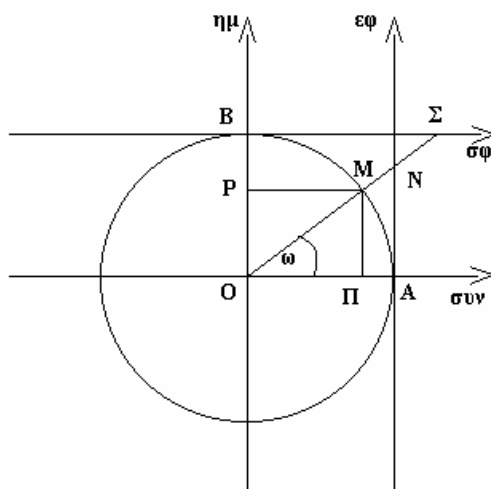
Αρμονική Πρόοδος.

- Ορισμός $(a_{v+1})^{-1}=(a_v)^{-1}+\omega$.
- β : αρμονικός μέσος των $a, \gamma \Leftrightarrow \beta=\frac{2a\gamma}{a+\gamma}$

Τριγωνομετρία.

Γενικά.

- Τριγωνομετρικός κύκλος - Τριγωνομετρικοί αριθμοί γωνίας.



$$\sigma\upsilon\nu\omega = \overline{O\Pi}, \quad \eta\mu\omega = \overline{O\text{P}}, \quad \epsilon\phi\omega = \overline{A\text{N}}, \quad \sigma\phi\omega = \overline{B\Sigma}.$$

- Μετατροπή μονάδων $\frac{\mu}{180} = \frac{\alpha}{\pi} = \frac{\beta}{200}$.
- Πρόσημο τριγωνομετρικών αριθμών

Τεταρτ ημόριο	ημ	συν	εφ	σφ
1 ^ο	+	+	+	+
2 ^ο	+	−	−	−
3 ^ο	−	−	+	+
4 ^ο	−	+	−	−

- Τριγωνομετρικοί αριθμοί βασικών τόξων.

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
	0°	30°	45°	60°	90°	180°	270°	360°
Ημx	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0	-1	0
συνx	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0	-1	0	1
Εφx	0	$\sqrt{3}/3$	1	$\sqrt{3}$	∞	0	∞	0
Σφx	∞	$\sqrt{3}$	1	$\sqrt{3}/3$	0	∞	0	∞

- Αναγωγή στο πρώτο τεταρτημόριο.

x	-θ	$(\pi/2)+\theta$	$(\pi/2)-\theta$	$\pi+\theta$	$(3\pi/2)+\theta$	$2\pi+\theta$
ημx	-ημθ	συνθ	συνθ	-ημθ	-συνθ	ημθ
συνx	συνθ	-ημθ	ημθ	-συνθ	ημθ	συνθ
εφx	-εφθ	-σφθ	σφθ	εφθ	-σφθ	εφθ
σφx	-σφθ	-εφθ	εφθ	σφθ	-εφθ	σφθ

- $\eta\mu^2 x + \sigma\upsilon\nu^2 x = 1.$
- $\epsilon\phi x = \frac{\eta\mu x}{\sigma\upsilon\nu x}.$
- $\sigma\phi x = \frac{\sigma\upsilon\nu x}{\eta\mu x}.$
- $\epsilon\phi x \cdot \sigma\phi x = 1.$

- $\eta\mu(\alpha + \beta) = \eta\mu\alpha \cdot \sigma\upsilon\nu\beta + \sigma\upsilon\nu\alpha \cdot \eta\mu\beta.$
- $\sigma\upsilon\nu(\alpha + \beta) = \sigma\upsilon\nu\alpha \cdot \sigma\upsilon\nu\beta - \eta\mu\alpha \cdot \eta\mu\beta.$
- $\epsilon\phi(\alpha + \beta) = \frac{\epsilon\phi\alpha + \epsilon\phi\beta}{1 - \epsilon\phi\alpha \cdot \epsilon\phi\beta}$
- $\sigma\phi(\alpha + \beta) = \frac{\sigma\phi\alpha \cdot \sigma\phi\beta - 1}{\sigma\phi\alpha + \sigma\phi\beta}$

- $\eta\mu(\alpha - \beta) = \eta\mu\alpha \cdot \sigma\upsilon\nu\beta - \sigma\upsilon\nu\alpha \cdot \eta\mu\beta.$
- $\sigma\upsilon\nu(\alpha - \beta) = \sigma\upsilon\nu\alpha \cdot \sigma\upsilon\nu\beta + \eta\mu\alpha \cdot \eta\mu\beta.$
- $\epsilon\phi(\alpha - \beta) = \frac{\epsilon\phi\alpha - \epsilon\phi\beta}{1 + \epsilon\phi\alpha \cdot \epsilon\phi\beta}$
- $\sigma\phi(\alpha - \beta) = \frac{\sigma\phi\alpha \cdot \sigma\phi\beta + 1}{\sigma\phi\alpha - \sigma\phi\beta}$

- $\eta\mu 2\alpha = 2\eta\mu\alpha \cdot \sigma\upsilon\nu\alpha = \frac{2\epsilon\phi\alpha}{1 + \epsilon\phi^2 \alpha}.$
- $\sigma\upsilon\nu 2\alpha = \begin{cases} \sigma\upsilon\nu^2 \alpha - \eta\mu^2 \alpha \\ 2\sigma\upsilon\nu\alpha \cdot \eta\mu\alpha \\ 1 - 2\eta\mu^2 \alpha \end{cases} = \frac{1 - \epsilon\phi^2 \alpha}{1 + \epsilon\phi^2 \alpha}$
- $\epsilon\phi 2\alpha = \frac{2\epsilon\phi\alpha}{1 - \epsilon\phi^2 \alpha}$
- $\sigma\phi 2\alpha = \frac{\sigma\phi^2 \alpha - 1}{2\sigma\phi\alpha} = \frac{1 - \epsilon\phi^2 \alpha}{2\epsilon\phi\alpha}$
- $\eta\mu\alpha = \pm \sqrt{\frac{1 - \sigma\upsilon\nu 2\alpha}{2}} = \pm \sqrt{\frac{\epsilon\phi^2 \alpha}{1 + \epsilon\phi^2 \alpha}}$
- $\sigma\upsilon\nu\alpha = \pm \sqrt{\frac{1 + \sigma\upsilon\nu 2\alpha}{2}} = \pm \sqrt{\frac{1}{1 + \epsilon\phi^2 \alpha}}$
- $\epsilon\phi\alpha = \pm \sqrt{\frac{1 - \sigma\upsilon\nu 2\alpha}{1 + \sigma\upsilon\nu 2\alpha}}$

- $\eta\mu 3\alpha = 3\eta\mu\alpha - 4\eta\mu^3\alpha$
- $\sigma\upsilon\nu 3\alpha = 4\sigma\upsilon\nu^3\alpha - 3\sigma\upsilon\nu\alpha$
- $\epsilon\varphi 3\alpha = \frac{3\epsilon\varphi\alpha - \epsilon\varphi^3\alpha}{1 - 3\epsilon\varphi^2\alpha}$
- $\sigma\varphi 3\alpha = \frac{\sigma\varphi^3\alpha - 3\sigma\varphi\alpha}{3\sigma\varphi^2\alpha - 1}$
- $2\eta\mu\alpha \cdot \sigma\upsilon\nu\beta = \eta\mu(\alpha + \beta) + \eta\mu(\alpha - \beta)$
- $2\sigma\upsilon\nu\alpha \cdot \sigma\upsilon\nu\beta = \sigma\upsilon\nu(\alpha + \beta) + \sigma\upsilon\nu(\alpha - \beta)$
- $2\eta\mu\alpha \cdot \eta\mu\beta = \sigma\upsilon\nu(\alpha - \beta) - \sigma\upsilon\nu(\alpha + \beta)$
- $\eta\mu\alpha + \eta\mu\beta = 2 \cdot \eta\mu \frac{\alpha + \beta}{2} \cdot \sigma\upsilon\nu \frac{\alpha - \beta}{2}$
- $\eta\mu\alpha - \eta\mu\beta = 2 \cdot \eta\mu \frac{\alpha - \beta}{2} \cdot \sigma\upsilon\nu \frac{\alpha + \beta}{2}$
- $\sigma\upsilon\nu\alpha + \sigma\upsilon\nu\beta = 2 \cdot \sigma\upsilon\nu \frac{\alpha + \beta}{2} \cdot \sigma\upsilon\nu \frac{\alpha - \beta}{2}$
- $\sigma\upsilon\nu\alpha - \sigma\upsilon\nu\beta = 2 \cdot \eta\mu \frac{\alpha + \beta}{2} \cdot \eta\mu \frac{\alpha - \beta}{2}$

Τριγωνομετρία.

Ταυτότητες για στοιχεία τριγώνου.

- $\epsilon\varphi A + \epsilon\varphi B + \epsilon\varphi \Gamma = \epsilon\varphi A \cdot \epsilon\varphi B \cdot \epsilon\varphi \Gamma.$
- $\eta\mu A + \eta\mu B + \eta\mu \Gamma = 4 \sigma\upsilon\nu \frac{A}{2} \cdot \sigma\upsilon\nu \frac{B}{2} \cdot \sigma\upsilon\nu \frac{\Gamma}{2}.$
- $\sigma\upsilon\nu A + \sigma\upsilon\nu B + \sigma\upsilon\nu \Gamma = 1 + 4 \cdot \eta\mu \frac{A}{2} \cdot \eta\mu \frac{B}{2} \cdot \eta\mu \frac{\Gamma}{2}.$
- $\eta\mu 2A + \eta\mu 2B + \eta\mu 2\Gamma = 4 \cdot \eta\mu A \cdot \eta\mu B \cdot \eta\mu \Gamma$
- $\sigma\upsilon\nu 2A + \sigma\upsilon\nu 2B + \sigma\upsilon\nu 2\Gamma = 1 - 4 \cdot \sigma\upsilon\nu A \cdot \sigma\upsilon\nu B \cdot \sigma\upsilon\nu \Gamma.$
- $\sigma\varphi \frac{A}{2} + \sigma\varphi \frac{B}{2} + \sigma\varphi \frac{\Gamma}{2} = \sigma\varphi \frac{A}{2} \cdot \sigma\varphi \frac{B}{2} \cdot \sigma\varphi \frac{\Gamma}{2}$
- $\sigma\varphi A \cdot \sigma\varphi B + \sigma\varphi B \cdot \sigma\varphi \Gamma + \sigma\varphi \Gamma \cdot \sigma\varphi A = 1.$
- $\epsilon\varphi \frac{A}{2} \cdot \epsilon\varphi \frac{B}{2} + \epsilon\varphi \frac{B}{2} \cdot \epsilon\varphi \frac{\Gamma}{2} + \epsilon\varphi \frac{\Gamma}{2} \cdot \epsilon\varphi \frac{A}{2} = 1.$
- $\frac{\alpha}{\eta\mu A} = \frac{\beta}{\eta\mu B} = \frac{\gamma}{\eta\mu \Gamma} = 2R. \quad (\text{Νόμος ημιτόνων.})$
- $\alpha^2 = \beta^2 + \gamma^2 - 2\beta\gamma \cdot \sigma\upsilon\nu A, \beta^2 = \gamma^2 + \alpha^2 - 2\gamma\alpha \cdot \sigma\upsilon\nu B, \gamma^2 = \alpha^2 + \beta^2 - 2\alpha\beta \cdot \sigma\upsilon\nu \Gamma$
(Νόμος συνημιτόνων.)

Τριγωνομετρία.**Τριγωνομετρικές εξισώσεις.**

- $\eta\mu x = \eta\mu \alpha \Leftrightarrow x = 2\kappa\pi + \alpha \text{ ή } x = (2\kappa + 1)\pi - \alpha, \kappa \in \mathbb{Z}.$
- $\sigma\upsilon\nu x = \sigma\upsilon\nu \alpha \Leftrightarrow x = 2\kappa\pi \pm \alpha, \kappa \in \mathbb{Z}.$
- $\epsilon\varphi x = \epsilon\varphi \alpha \Leftrightarrow x = \kappa\pi + \alpha, \kappa \in \mathbb{Z}.$
- $\sigma\varphi x = \sigma\varphi \alpha \Leftrightarrow x = \kappa\pi + \alpha, \kappa \in \mathbb{Z}.$

Λογάριθμοι.

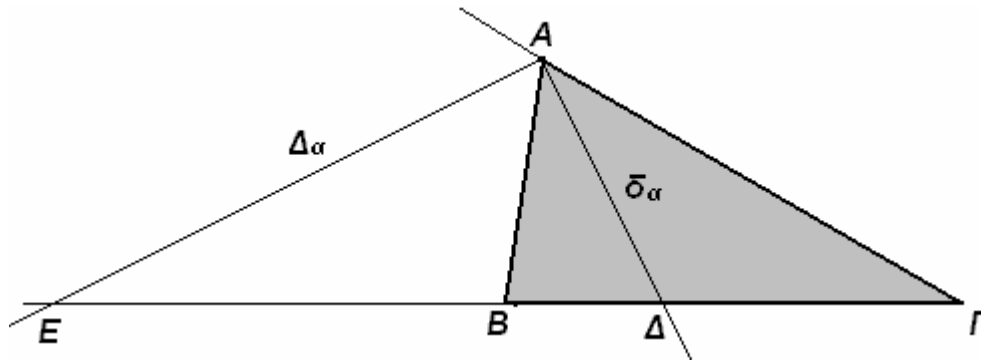
- $\log_a(x \cdot y) = \log_a x + \log_a y, \forall x > 0, \forall y > 0.$
- $\log_a(x : y) = \log_a x - \log_a y, \forall x > 0, \forall y > 0.$
- $\log_a(x^v) = v \cdot \log_a x, \forall x > 0 \text{ και } v \in \mathbb{N}.$
- $\log x = \log_{10} x.$
- $\ln x = \log_e x$
- $\log_a x = \frac{\log_b x}{\log_b a}$

Συνδυαστική.

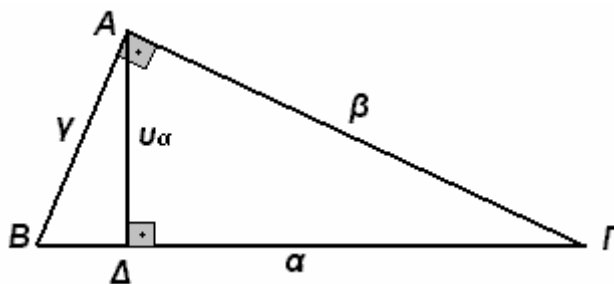
- Μεταθέσεις των v στοιχείων : $M_v = v!.$
- Διατάξεις των μ στοιχείων σε v θέσεις : $\Delta_v^\mu = \frac{\mu!}{(\mu - v)!}$
- Διατάξεις με επανάληψη των μ στοιχείων σε v θέσεις : $E_v^\mu = \mu^v.$
- Συνδυασμοί των v στοιχείων ανά κ : $\binom{v}{\kappa} = \frac{v!}{\kappa!(v - \kappa)!}$

Πιθανότητες.

- $P(A) = \frac{N(A)}{N(\Omega)}.$
 - $0 \leq P(A) \leq 1.$
 - $P(\Omega) = 1.$
 - $P(\emptyset) = 0.$
 - $A \subseteq B \Rightarrow P(A) \leq P(B)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B).$
- $P(A') = 1 - P(A).$
- $P(B | A) = \frac{P(A \cap B)}{P(A)}.$

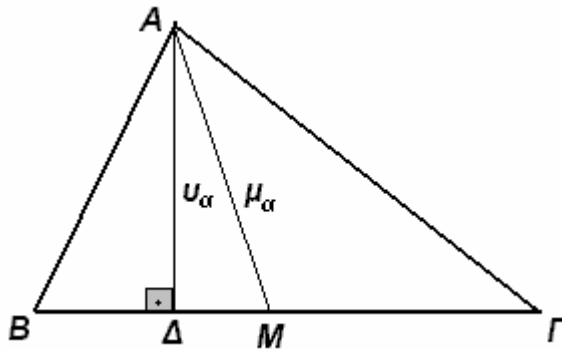


- $\frac{\Delta B}{\Delta \Gamma} = \frac{EB}{E\Gamma} = \frac{AB}{A\Gamma}$
- $B\Delta = \frac{\alpha\gamma}{\beta + \gamma}$ • $\Delta\Gamma = \frac{\alpha\beta}{\beta + \gamma}$ • $BE = \frac{\alpha\gamma}{\beta - \gamma}$ • $E\Gamma = \frac{\alpha\beta}{\beta - \gamma}$
- Τα E και Δ λέγονται αρμονικά συζυγή των B και Γ.
- Τα A, B, Γ, και Δ λέγονται αρμονική τετράδα.



Τα τρίγωνα ABΓ,
ΔBA και ΔAΓ
είναι όμοια.

- $\gamma^2 = \alpha \cdot B\Delta$ και $\beta^2 = \alpha \cdot \Gamma\Delta$.
- $\alpha^2 = \beta^2 + \gamma^2$. (Πυθαγόρειο Θεώρημα.)
- $u_\alpha = B\Delta \cdot \Delta\Gamma$.
- $\beta \cdot \gamma = \alpha \cdot u_\alpha$.
- $\frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{1}{u_\alpha^2}$

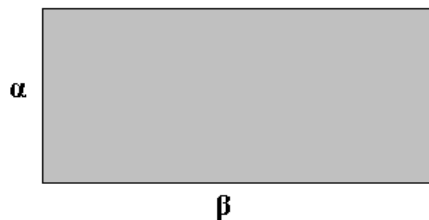
Γεωμετρία.**Μετρικές σχέσεις σε τυχαίο τρίγωνο.**

- $ΑΓ^2 = ΑΒ^2 + ΒΓ^2 - 2 \cdot ΒΓ \cdot ΒΔ$, αν η γωνία $\hat{B}$ είναι οξεία..
- $ΑΓ^2 = ΑΒ^2 + ΒΓ^2 + 2 \cdot ΒΓ \cdot ΒΔ$, αν η γωνία $\hat{B}$ είναι αμβλεία..
- $ΒΔ = \frac{\gamma^2 + \alpha^2 - \beta^2}{2 \cdot \alpha}$
- $u_\alpha = \frac{2 \cdot \sqrt{\tau \cdot (\tau - \alpha) \cdot (\tau - \beta) \cdot (\tau - \gamma)}}{\alpha}$.
- $\beta^2 + \gamma^2 = 2 \cdot \mu_\alpha^2 + \frac{\alpha^2}{2}$. (1^ο Θεώρημα διαμέσων.)
- $\beta^2 + \gamma^2 = 2 \cdot \alpha \cdot ΜΔ$.
- $\mu_\alpha^2 = \frac{2\beta^2 + 2\gamma^2 - \alpha^2}{4}$.
- $\frac{1}{v_\alpha} + \frac{1}{v_\beta} + \frac{1}{v_\gamma} = \frac{1}{\rho}$.

Γεωμετρία**Κανονικά πολύγωνα.**

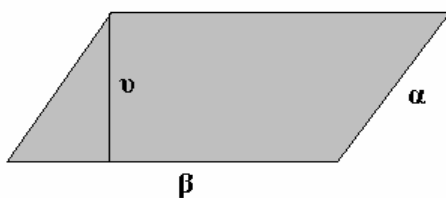
- | | | | |
|---|------------------------------------|---|--|
| • $\omega_v = \frac{360^\circ}{v}$ | • $\phi_v = 180^\circ - \omega_v$ | • $\left(\frac{\lambda_v}{2}\right) + \alpha_v^2 = R^2$ | • $E_v = \frac{v}{2} \lambda_v \alpha_v$ |
| • $\lambda_{2v} = \sqrt{2R^2 - R\sqrt{4R^2 - \lambda_v^2}}$ | | • $\lambda_3 = R\sqrt{3}$ | • $\alpha_3 = \frac{R}{2}$ |
| • $\lambda_4 = R\sqrt{2}$ | • $\alpha_4 = \frac{R\sqrt{2}}{2}$ | • $\lambda_6 = R$ | • $\alpha_6 = \frac{R\sqrt{3}}{2}$ |
| • | • | • | • |

- Ορθογώνιο.



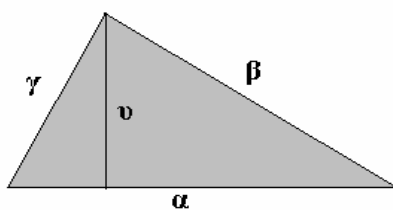
$$E = \alpha \cdot \beta$$

- Παραλληλόγραμμο.



$$E = \beta \cdot \upsilon$$

- Τρίγωνο.



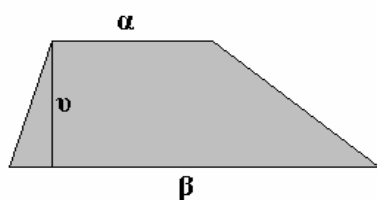
$$E = \frac{\beta \cdot \upsilon}{2}$$

$$E = \sqrt{\tau \cdot (\tau - \alpha) \cdot (\tau - \beta) \cdot (\tau - \gamma)}$$

$$E = \tau \cdot \rho$$

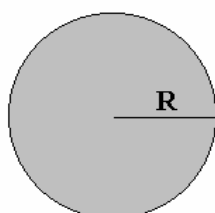
$$E = \frac{\alpha \cdot \beta \cdot \gamma}{4R}$$

- Τραπεζίιο.



$$E = \frac{\alpha + \beta}{2} \cdot \upsilon$$

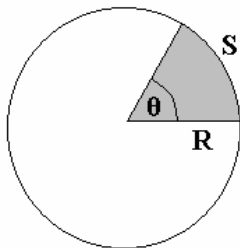
- Κύκλος.



$$E = \pi R^2$$

$$\Gamma = 2\pi R$$

- Κυκλικός τομέας - τόξο.



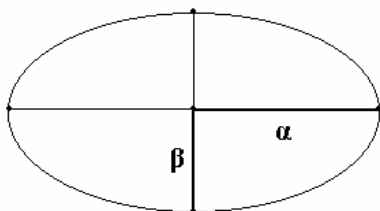
$$S = \theta \cdot R, \theta \text{ ακτίνια.}$$

$$S = \frac{\theta}{360} 2\pi R, \theta \text{ μοίρες.}$$

$$E = \frac{1}{2} \theta R^2, \theta \text{ ακτίνια.}$$

$$E = \frac{\theta}{360} \pi R^2, \theta \text{ μοίρες.}$$

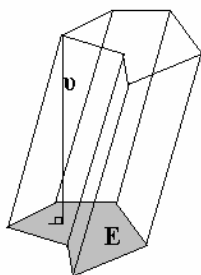
- Έλλειψη.



$$E = \pi \cdot \alpha \cdot \beta.$$

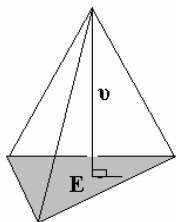
$$\Gamma \approx \sqrt{\frac{1}{2}(\alpha^2 + \beta^2)}$$

- Πρίσμα.



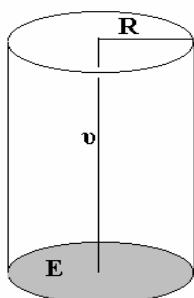
$$V = E_{\beta} \cdot v.$$

- Πυραμίδα



$$V = \frac{1}{3} \cdot E_{\beta} \cdot v.$$

- Κύλινδρος.

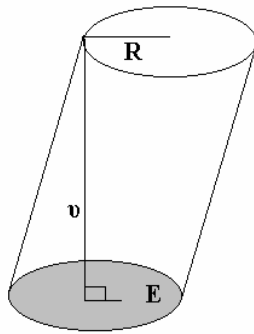


$$E_{\pi} = 2\pi R \cdot v.$$

$$E_{\text{ολ}} = 2\pi R(R + v).$$

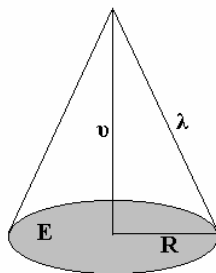
$$V = \pi R^2 v.$$

• Πλάγιος κύλινδρος



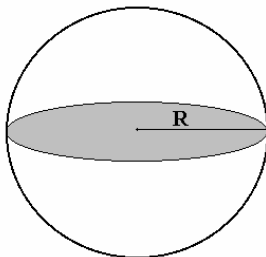
$$V = \pi R^2 \cdot v.$$

• Κώνος.



$$V = \frac{1}{3} \cdot \pi R^2 \cdot v.$$

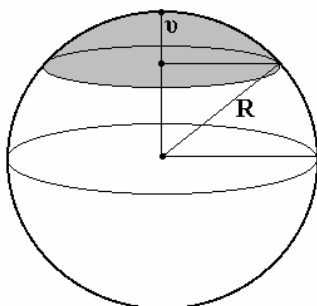
• Σφαίρα.



$$E = 4 \cdot \pi R^2.$$

$$V = \frac{4}{3} \cdot \pi R^3.$$

• Κυκλικό τμήμα.



$$E = 2\pi R v.$$

$$V = \frac{1}{3} \cdot \pi v^2 (3R - v).$$

Ανάλυση**Όρια.**

Όταν υπάρχουν τα όρια των συναρτήσεων f και g τότε ισχύουν

• $\lim_{x \rightarrow \sigma} f(x) = \ell \Leftrightarrow$ ○ $\Leftrightarrow \lim_{x \rightarrow \sigma} [f(x) - \ell] = 0 \Leftrightarrow$ ○ $\Leftrightarrow \lim_{x \rightarrow \sigma} (-f(x)) = -\ell \Leftrightarrow$ • $\lim_{x \rightarrow \sigma} f(x) = \ell \Rightarrow$ $\Rightarrow \frac{ \ell }{2} < f(x) < \frac{3 \ell }{2}$ • $\lim_{x \rightarrow \sigma} [\lambda \cdot f(x)] = \lambda \cdot \lim_{x \rightarrow \sigma} f(x)$	• $\lim_{x \rightarrow \sigma} (f(x) + g(x)) = \lim_{x \rightarrow \sigma} f(x) + \lim_{x \rightarrow \sigma} g(x)$ • $\lim_{x \rightarrow \sigma} (f(x) - g(x)) = \lim_{x \rightarrow \sigma} f(x) - \lim_{x \rightarrow \sigma} g(x)$ • $\lim_{x \rightarrow \sigma} (f(x) \cdot g(x)) = \lim_{x \rightarrow \sigma} f(x) \cdot \lim_{x \rightarrow \sigma} g(x)$ • $\lim_{x \rightarrow \sigma} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow \sigma} f(x)}{\lim_{x \rightarrow \sigma} g(x)} \quad \left\{ \begin{array}{l} \text{Εφόσον} \\ \text{ορίζονται καλώς} \\ \text{τα κλάσματα..} \end{array} \right.$ • $\lim_{x \rightarrow \sigma} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow \sigma} f(x)} \quad \left\{ \begin{array}{l} \text{Εφόσον} \\ \text{ορίζονται} \\ \text{καλώς οι ρίζες.} \end{array} \right.$
--	--

Ανάλυση**Παράγωγοι.**

• $[c]' = 0$ • $[x]' = 1$ • $[x^v]' = vx^{v-1}$ • $[x^p]' = px^{p-1}$ • $[\eta \mu x]' = \sigma \nu x$ • $[\sigma \nu x]' = -\eta \mu x$	• $[e^x]' = e^x$ • $[\ln x]' = \frac{1}{x}$ • $[\varepsilon \varphi x]' = \frac{1}{\sigma \nu x^2}$ • $[\sigma \varphi x]' = \frac{-1}{\eta \mu^2 x}$	• $[f + g]' = f' + g'$ • $[f - g]' = f' - g'$ • $[f \cdot g]' = f' \cdot g + f \cdot g'$ • $\left[\frac{f}{g}\right]' = \frac{f' \cdot g - f \cdot g'}{g^2}$ • $[\lambda \cdot f]' = \lambda \cdot f'$ • $[f^v]' = v \cdot f^{v-1} \cdot f'$
---	--	---

- $[f(g(x))]' = f'(g(x)) \cdot g'(x) \quad \eta \quad \frac{df(g(x))}{dx} = \frac{df(g(x))}{dg(x)} \cdot \frac{dg(x)}{dx}$

• $\int_a^\beta f(x)dx := \lim \left[\frac{\beta - \alpha}{v} \cdot \sum_{\kappa=1}^v f \left(\alpha + \kappa \cdot \frac{\beta - \alpha}{v} \right) \right]$ • $\int_a^\beta [f(x) + g(x)]dx = \int_a^\beta f(x)dx + \int_a^\beta g(x)dx$ • $\int_a^\beta \lambda \cdot f(x)dx = \lambda \cdot \int_a^\beta f(x)dx$ • $\int_a^\beta f(x)dx = \int_a^\gamma f(x)dx + \int_\gamma^\beta f(x)dx$ • $\min f \cdot (\beta - \alpha) \leq \int_a^\beta f(x)dx \leq \max f \cdot (\beta - \alpha)$ • $\int_a^\beta f'(x) \cdot g(x)dx = [f(x) \cdot g(x)]_a^\beta - \int_a^\beta f(x) \cdot g'(x)dx$ • $\int_a^\beta f(g(x)) \cdot g'(x)dx = \int_{g(a)}^{g(\beta)} f(y)dy$	• $\int_a^a f(x)dx = 0$ • $\int_\beta^a f(x)dx = -\int_a^\beta f(x)dx$ • $\int_a^\beta f'(x)dx = f(\beta) - f(\alpha)$ • $\left[\int_a^x f(t)dt \right]' = f(x)$ • $\left[\int_a^{g(x)} f(t)dt \right]' = f(g(x)) \cdot g'(x)$ • $\bar{f} = \frac{1}{\beta - \alpha} \cdot \int_a^\beta f(x)dx$
--	--

• $\int f'(x)dx = f(x) + c$ • $\int 1dx = x + c$ • $\int \frac{1}{x} dx = \ln x + c$ • $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + c$	• $\int \sigma \nu x dx = \eta \mu x + c$ • $\int \eta \mu x dx = -\sigma \nu x + c$ • $\int \frac{1}{\sigma \nu^2 x} dx = \varepsilon \varphi x + c$ • $\int \frac{1}{\eta \mu^2 x} dx = -\sigma \varphi x + c$	• $\int e^x dx = e^x + c$ • $\int a^x dx = \frac{a^x}{\ln a} + c$
---	---	--



ΠΙΝΑΚΕΣ ΤΡΙΓΩΝΟΜΕΤΡΙΚΩΝ ΑΡΙΘΜΩΝ

ω	ημω	συνω	εφω	σφω	ω	ημω	συνω	εφω	σφω
0 °	0,0000000	1,0000000	0,0000000		45 °	0,7071068	0,7071068	1,0000000	1,0000000
1 °	0,0174524	0,9998477	0,0174551	57,2899616	46 °	0,7193398	0,6946584	1,0355303	0,9656888
2 °	0,0348995	0,9993908	0,0349208	28,6362533	47 °	0,7313537	0,6819984	1,0723687	0,9325151
3 °	0,0523360	0,9986295	0,0524078	19,0811367	48 °	0,7431448	0,6691306	1,1106125	0,9004040
4 °	0,0697565	0,9975641	0,0699268	14,3006663	49 °	0,7547096	0,6560590	1,1503684	0,8692867
5 °	0,0871557	0,9961947	0,0874887	11,4300523	50 °	0,7660444	0,6427876	1,1917536	0,8390996
6 °	0,1045285	0,9945219	0,1051042	9,5143645	51 °	0,7771460	0,6293204	1,2348972	0,8097840
7 °	0,1218693	0,9925462	0,1227846	8,1443464	52 °	0,7880108	0,6156615	1,2799416	0,7812856
8 °	0,1391731	0,9902681	0,1405408	7,1153697	53 °	0,7986355	0,6018150	1,3270448	0,7535541
9 °	0,1564345	0,9876883	0,1583844	6,3137515	54 °	0,8090170	0,5877853	1,3763819	0,7265425
10 °	0,1736482	0,9848078	0,1763270	5,6712818	55 °	0,8191520	0,5735764	1,4281480	0,7002075
11 °	0,1908090	0,9816272	0,1943803	5,1445540	56 °	0,8290376	0,5591929	1,4825610	0,6745085
12 °	0,2079117	0,9781476	0,2125566	4,7046301	57 °	0,8386706	0,5446390	1,5398650	0,6494076
13 °	0,2249511	0,9743701	0,2308682	4,3314759	58 °	0,8480481	0,5299193	1,6003345	0,6248694
14 °	0,2419219	0,9702957	0,2493280	4,0107809	59 °	0,8571673	0,5150381	1,6642795	0,6008606
15 °	0,2588190	0,9659258	0,2679492	3,7320508	60 °	0,8660254	0,5000000	1,7320508	0,5773503
16 °	0,2756374	0,9612617	0,2867454	3,4874144	61 °	0,8746197	0,4848096	1,8040478	0,5543091
17 °	0,2923717	0,9563048	0,3057307	3,2708526	62 °	0,8829476	0,4694716	1,8807265	0,5317094
18 °	0,3090170	0,9510565	0,3249197	3,0776835	63 °	0,8910065	0,4539905	1,9626105	0,5095254
19 °	0,3255682	0,9455186	0,3443276	2,9042109	64 °	0,8987940	0,4383711	2,0503038	0,4877326
20 °	0,3420201	0,9396926	0,3639702	2,7474774	65 °	0,9063078	0,4226183	2,1445069	0,4663077
21 °	0,3583679	0,9335804	0,3838640	2,6050891	66 °	0,9135455	0,4067366	2,2460368	0,4452287
22 °	0,3746066	0,9271839	0,4040262	2,4750869	67 °	0,9205049	0,3907311	2,3558524	0,4244748
23 °	0,3907311	0,9205049	0,4244748	2,3558524	68 °	0,9271839	0,3746066	2,4750869	0,4040262
24 °	0,4067366	0,9135455	0,4452287	2,2460368	69 °	0,9335804	0,3583679	2,6050891	0,3838640
25 °	0,4226183	0,9063078	0,4663077	2,1445069	70 °	0,9396926	0,3420201	2,7474774	0,3639702
26 °	0,4383711	0,8987940	0,4877326	2,0503038	71 °	0,9455186	0,3255682	2,9042109	0,3443276
27 °	0,4539905	0,8910065	0,5095254	1,9626105	72 °	0,9510565	0,3090170	3,0776835	0,3249197
28 °	0,4694716	0,8829476	0,5317094	1,8807265	73 °	0,9563048	0,2923717	3,2708526	0,3057307
29 °	0,4848096	0,8746197	0,5543091	1,8040478	74 °	0,9612617	0,2756374	3,4874144	0,2867454
30 °	0,5000000	0,8660254	0,5773503	1,7320508	75 °	0,9659258	0,2588190	3,7320508	0,2679492
31 °	0,5150381	0,8571673	0,6008606	1,6642795	76 °	0,9702957	0,2419219	4,0107809	0,2493280
32 °	0,5299193	0,8480481	0,6248694	1,6003345	77 °	0,9743701	0,2249511	4,3314759	0,2308682
33 °	0,5446390	0,8386706	0,6494076	1,5398650	78 °	0,9781476	0,2079117	4,7046301	0,2125566
34 °	0,5591929	0,8290376	0,6745085	1,4825610	79 °	0,9816272	0,1908090	5,1445540	0,1943803
35 °	0,5735764	0,8191520	0,7002075	1,4281480	80 °	0,9848078	0,1736482	5,6712818	0,1763270
36 °	0,5877853	0,8090170	0,7265425	1,3763819	81 °	0,9876883	0,1564345	6,3137515	0,1583844
37 °	0,6018150	0,7986355	0,7535541	1,3270448	82 °	0,9902681	0,1391731	7,1153697	0,1405408
38 °	0,6156615	0,7880108	0,7812856	1,2799416	83 °	0,9925462	0,1218693	8,1443464	0,1227846
39 °	0,6293204	0,7771460	0,8097840	1,2348972	84 °	0,9945219	0,1045285	9,5143645	0,1051042
40 °	0,6427876	0,7660444	0,8390996	1,1917536	85 °	0,9961947	0,0871557	11,4300523	0,0874887
41 °	0,6560590	0,7547096	0,8692867	1,1503684	86 °	0,9975641	0,0697565	14,3006663	0,0699268
42 °	0,6691306	0,7431448	0,9004040	1,1106125	87 °	0,9986295	0,0523360	19,0811367	0,0524078
43 °	0,6819984	0,7313537	0,9325151	1,0723687	88 °	0,9993908	0,0348995	28,6362533	0,0349208
44 °	0,6946584	0,7193398	0,9656888	1,0355303	89 °	0,9998477	0,0174524	57,2899616	0,0174551
45 °	0,7071068	0,7071068	1,0000000	1,0000000	90 °	1,0000000	0,0000000	#####	0,0000000